Terminal Voltage & Internal Resistance

In a cell, chemical reactions cause a charge separation to occur between the two terminals. This potential difference is called the *Electromotive Force* (the term is misleading; it is not a force), often referred to as *EMF*, with symbol 'ε' and measured in Volts.

EMF = the voltage across a supply when there is no current flowing.

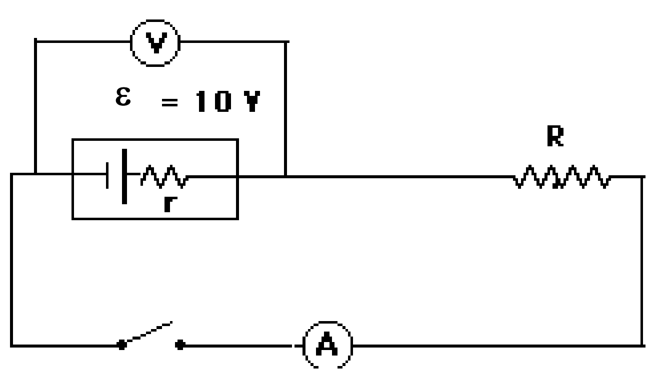
V_T (terminal voltage) = the *actual* potential difference across the terminals of the supply when a current is being supplied.

When a closed circuit is set up so that electrons flow from negative to positive terminals, the terminal voltage drops below **EMF** value. Here's why:

- Chemical reactions within the cell cannot separate charges fast enough to keep maximum charge separation.
- The charges must flow between electrolyte and terminals, and there is always some resistance to this, called *internal resistance* (**r**).
- As a result, when current flows, there is an internal voltage drop equal to **ir**, and from this,

$$V_T = \varepsilon - Ir \qquad \rightarrow \text{ note that when } I = 0,\; V_T = \varepsilon$$

Examine the following schematic. Note that the small box represents the battery. The terminal voltage V_T is measured from one end of the box to the other, and is read by the voltmeter. To find the internal resistance **r** of the power supply, perform the following steps:



- find the potential difference with the switch open; this is the **EMF**. Note that the voltmeter can't do this (it draws some current), so other techniques must be used.
- close switch and measure the potential difference with current flowing **(V_T)** and measure current with the ammeter.

EMF - V_T = lost voltage (Ir) → **$\varepsilon - V_T = Ir$**

→ so **$r = (\varepsilon - V_T)/I$**

We can also use use internal and external resistance to find the current supplied by a power source. Start with

TV = ε - Ir → **$\varepsilon = V_T + Ir = IR_o + Ir$**

→ which becomes **$I = \varepsilon/(R_o + r)$** where **$R_o$** is the total external resistance of the circuit.

Example #17: When a 6.0 V EMF battery was connected to a 15 Ω resistance, a current of 375 mA occurred and the voltmeter reading was 5.625 V.

(a) Find the internal resistance r of this supply.

(b) If this battery is now connected to a 5.0 Ω resistor, what current will flow?

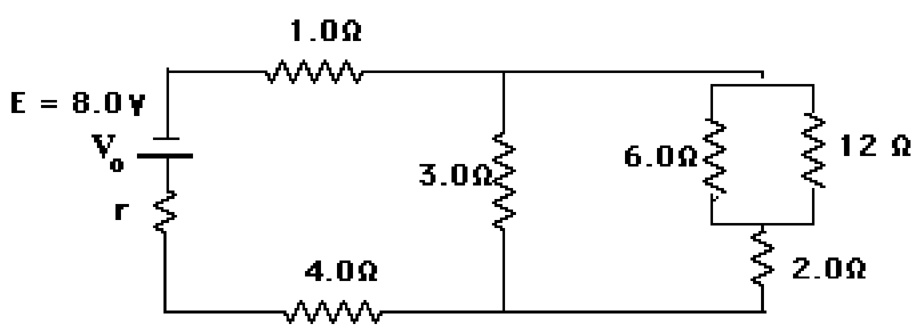
(see Circuitry Ex 17 for answer)

Example #18: A battery of EMF 8.0 V and internal resistance r = 1.0 Ω is connected to an external circuit as shown. Find:

(a) the equivalent resistance of the circuit.

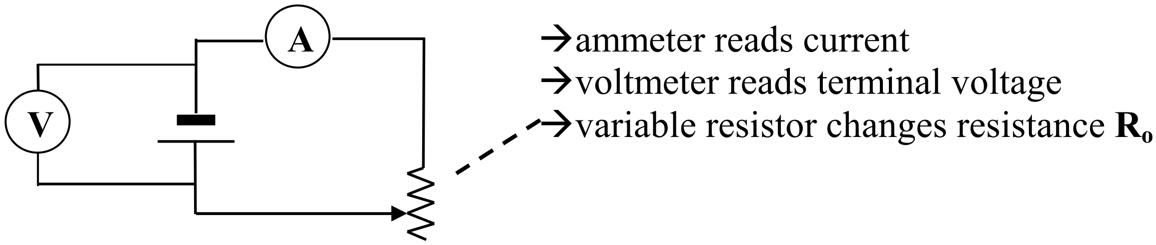
(b) the total current leaving the battery.

(c) the potential difference between the terminals of the battery.

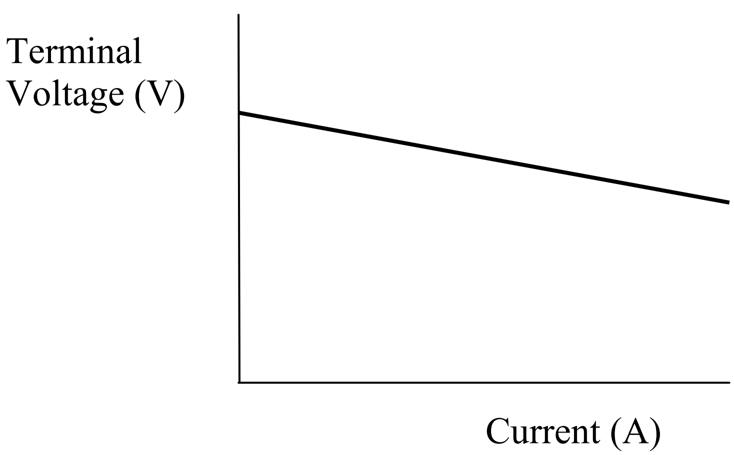


(see Circuitry Ex 18 for answer)

Note that the equation $\mathbf{V_T = \varepsilon - Ir}$ can be derived from graphing. Using a variable resistor in the following set-up:



As you would expect, changing the external resistance $\mathbf{R_o}$ in the circuit changes the current drawn; i.e., more resistance means less current, and vice versa. However, what you might *not* expect is that the voltage reading $\mathbf{V_T}$ *also* changes, due to the changing current. Graphing terminal voltage vs. current:



As current increases, terminal voltage slowly decreases in a linear manner. Using the standard graphing equation $y = kx + b$, we obtain

$$\mathbf{V_T = -\mathit{k}I + \mathit{b}} \rightarrow \text{where } \boldsymbol{k} \text{ is a negative slope}$$

Compare this equation with $\mathbf{V_T = \varepsilon - Ir}$, we see:

- slope $\boldsymbol{k}$ = internal resistance **r**
- y-intercept $\boldsymbol{b}$ = **EMF** $\boldsymbol{\varepsilon}$

This technique is commonly used to find the **EMF** of any battery or cell.

Two last points:
Consider a small 1.5 V cell connected to a closed circuit with another, much larger power supply that moves electrons from positive to negative terminals in the small cell. In this case, $\mathbf{V_T}$ is greater than the **EMF** value, due to the opposite-to-normal flow of current; as a result,

$$\mathbf{V_T = \varepsilon + Ir}$$

In some batteries, this reversed current will cause the chemical reactions to be reversed, effectively *recharging* the batteries; in other cases, the reactions cannot be reversed due to the chemicals involved.

Finally, a complete circuit without an external resistance $\mathbf{R_o}$ is called a *short circuit*. Only the internal resistance (**r**) of the supply limits the flow of current in a short circuit.